

Informed or Not: The Distorted Treatment of Applied Mathematics in Math Curricula and
Its Effect on Students

by

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INTRODUCTION

The call for more problem solving in United States math classrooms has taken many forms and come from many outlets (National Research Council, 1989; NCTM, 1989; Ball, 2003). The motives vary but the National Commission on Mathematics and Science Teaching for the 21st Century summarizes a perennial concern for our economic and political future:

It is hard to imagine that students in these classes are gaining the conceptual and problem-solving skills they need to function effectively as workers and citizens in today's world—a world that increasingly depends on mathematics and science (2000, p. 21).

The definition of “problem solving” is complicated, however. It may ultimately be easier to define problem solving by contradiction: problem solving is *not* the application of one memorized procedure to an exercise in the pursuit of a single numerical answer. The implementation of problem solving in the classroom is complicated. The road that takes a mandate from any governing bodies to the work a student does in a classroom is perilous. Hennigsen and Stein (1997) and Doyle (1998) examined the interaction between teachers and students and identified classroom-level factors that contribute to the decline of a problem solving task. This paper will, instead, study the features of a problem-solving task as it was designed by a publisher and consider their implications for the work a student will do in the classroom.

This paper will consider the sub-genre of *applied* mathematical problem solving — mathematics that is applied outside of the math classroom — and two complicated facets of its implementation, one cognitive (how students *think* about applied problem solving) and one affective (how students *feel* about applied problem solving).

Publishers of math curricula often interpret these challenging mandates for applied mathematical problem solving in ways that trivialize them, resulting in tasks that are accurately described by Boaler's term “pseudocontext”:

... contexts that students were meant to believe but for which they should not use any of their real-world knowledge. (2008, p. 43)

Pseudocontext also results when a student's real-world knowledge includes the awareness that *math doesn't belong in this context*, as in this example where the New York State Regents have contrived a pseudocontext for operations with complex numbers (The University of the State of New York, 2001):

Melissa and Joe are playing a game with complex numbers. If Melissa has a score of $5 - 4i$ and Joe has a score of $3 + 2i$, what is their total score?

1. $8 + 6i$
2. $8 + 2i$
3. $8 - 6i$
4. $8 - 2i$

Even when the implementations of applied mathematical problem solving aren't contrived, they often feature a curious distortion of the practice of applied mathematicians. Consider an example drawn from *Heath Algebra 1: An Integrated Approach*:

A basketball court is 46 feet wide and 84 feet long. The court has a border that is 10 feet wide. The cost of a hardwood floor is \$9 per square foot. How much will it cost to put a new floor on both the court and its border? (Larson, Kanold, & Stiff, 2006, p. 3.)

Notice that students are given the goal of the problem and the information essential to *completing* the goal of the problem at the same time. Students are told the dimensions of the basketball court and the cost of hardwood and they're also told their goal is to find the cost of a new floor. Furthermore, they are given no more and no less information than what is needed for the resolution of that goal. In applied mathematics

outside of textbooks, it is more often the case that a problem solver has a goal and must decide for herself what information will be helpful and what information will be unhelpful.

The treatment of applied mathematics in textbooks is, therefore, unnatural. It is a distortion of the practice of applied mathematicians. Teaching comprises many unnatural acts, though (Jackson, 1986; Murray, 1989). For example, teachers ask questions to which they already know the answer. They elicit errors and provoke disequilibrium when it might feel more natural to protect students from error and maintain equilibrium. This distortion of applied problem solving is no more unnatural than any of teaching's other unnatural acts. Those other unnatural acts, though, have their rationale in improved conditions for student learning. Teachers ask questions to which they already know the answer because they want to know what their students think. Teachers elicit errors so that they may be addressed. The cognitive aspect of this study investigates how the distorted treatment of applied mathematics in textbooks affects conditions for student problem solving.

The affective dimension of this study spans several axes. We know that a student's confidence in her ability to solve a problem correlates with her ability to solve that problem correctly (Schoenfeld, 1985; Fennema & Sherman, 1976; Pajares & Graham, 1999). We know it's important that students understand the goal of the work they are assigned, the problem's "unknown" (Polya, 1945, p. 6). We also know it's important that students *enjoy* their work. It is unknown how the distorted treatment of applied mathematics in textbooks affects these three affective variables.

RESEARCH QUESTIONS

Summarizing my two research questions then:

1. *Cognitive*. How does the information given in the text of an applied math problem affect the problem solving strategies of students? Is it consequential? If so, does it help or hinder them?

2. *Affective*. How does the information given in the text of an applied math problem affect a student's confidence in her ability to solve the problem, her understanding of the goal of that problem, and her enjoyment of that problem?

PARTICIPANTS

To answer these research questions, I asked eight English student teachers at a private research university in the San Francisco Bay Area to solve one of two versions of the same high school geometry problem. I selected adults rather than students in an attempt to establish a baseline for background mathematical knowledge. These student teachers had each met the math requirement for undergraduate and graduate studies. At one point they were each proficient in the domain of my study: high school geometry. I selected *English* student teachers rather than *math* student teachers because math student teachers, I theorized, are so experienced (and likely proficient) at the kind of problems I'm investigating they'd be insensitive to differences between the control and experimental treatments.

The student teachers' math backgrounds all included high school geometry but were surprisingly diverse beyond that course. I expected that many would have opted out of mathematical studies once they were no longer required, but one participant had studied differential equations just four years earlier in his undergraduate education and another had studied multi-variable calculus in high school, five years earlier. Several others reported studying statistics to fulfill the requirements of their undergraduate majors after which they stopped taking math courses altogether. One student teacher last studied math sixteen years earlier, a geometry class, in her sophomore year of high school.

METHODS

I asked the student teachers to volunteer in pairs. There was one male pair and three female pairs. (Their entire cohort comprised three males and thirteen females, for comparison.) I assigned each pair randomly to either the control or experimental group.

I drew on the designs of previous researchers who manipulated the content of word problems (Fayol, et al., 1987; Thevenot, et al., 2007). Thevenot, et al., for instance, studied the difference in the achievement of French fourth graders on a series of problems that featured the question at either the beginning or the end of the text. They found positive and significant effects when the question was placed at the beginning of the text, rather than the end. They found the strongest effect with lower-achieving students.

In this study, both the control and experimental groups saw the picture in Figure 1 of a water tank being filled by a hose:

Figure 1: A frame from the video that launched both problems.



The control group saw this text beneath the image:

The water tank is in the shape of an octagonal prism. The height of the tank is 36.5 centimeters. The side length of the regular octagon is 12.4 centimeters. The flow rate of the hose is 1.89 fluid ounces per second. There are 29.57 cubic

centimeters in 1 fluid ounce. How long will it take the hose to fill up the water tank? If you need help with information or procedures, you may ask for it.

The experimental group saw this text beneath the image, which is identical to the last two sentences of the control treatment:

How long will it take the hose to fill up the water tank? If you need help with information or procedures, you may ask for it.

It would have been impossible for the experimental group to resolve the problem to a numerical answer without asking me for information about the context. Still, before both groups began work, I reiterated that I could help them with information about the problem if they needed it. I also volunteered help with procedures if they had forgotten them. I didn't want to become an auxiliary member of the problem solving pairs, however, so I constrained my help in the following ways:

1. When asked for information, I pretended I had a tape measure. I could measure linear distances between points that were precisely defined in the space of the problem. I could measure and report distances from edge to edge and vertex to vertex, for example, but I couldn't measure to or from any point the problem solvers couldn't precisely define for me. (eg. The octagon's center lacks a visual referent. It's invisible. If the participants told me how to locate the center by finding the intersection of different diagonals, I would measure to and from it, however.) I also offered definitions of the sort that could be easily found in a glossary. (eg. "A regular polygon has congruent sides and angles.") See Appendix A for the complete list of information I allowed myself to offer the participants.
2. When asked for procedures, I made available formulae and relationships that would have been clearly defined in a textbook. (eg. "To find the volume of a prism, multiply the area of its base by its height." See Appendix B for the

complete list.) None of these procedures referred to the context of the water tank. They all referred to abstract figures and shapes.

Further codifying the help I could offer, I wrote the information and procedures down in advance on index cards, which I developed using the two criteria above and refined through piloting.

Essentially, through these constraints, I put my participants in the position of someone who owned a) that water tank, b) a tape measure, and c) a geometry textbook, someone who was wondering how long it would take to fill the tank up with a hose. I didn't help the participants develop a solution strategy. I didn't tell them *when* to use a particular procedure. And when they used a given procedure, I didn't tell them if they had implemented the procedure correctly or not.

For the most part, I was able to apply these constraints without complication. The control group didn't ask for any information beyond that which was provided in the text of their problem. One experimental pair asked me for the volume of the tank. I replied that I didn't have an instrument for measuring volume but that I could measure other distances for them. Another pair asked me for the distance from the center of the octagon to an edge but they hadn't precisely defined where I could find the center so I said they'd have to use math to find it.

Cognitive data.

To determine how the different treatments of the problem affected the participants' problem solving, I recorded audio and video of each session. I emphasized that the pairs should work together and talk together about the problem (Ericsson & Simon, 1993).

I maintained a content log of the participants' sessions and recorded, specifically, when participants asked for a given information card, when participants asked for a procedure card, and the events immediately preceding those requests (Appendix C). I recorded in the content log whether or not each pair had solved the problem correctly. When relevant, I transcribed the content log.

At the end of the session, I interviewed each pair. As mentioned, I tried to determine a baseline for the participants' experience with applied mathematical problem solving by asking them to tell me the last math class they took and when they took it. I also asked them, "What kind of problem did you expect when you were told you would be solving an applied high school geometry problem?"

I also timed the duration of each pair's problem solving session and whether or not their answer was correct.

Affective data.

To determine how the different treatments affected the participants' confidence in their ability to solve the problem, I sampled their confidence at different intervals throughout their session. I asked them a question derived from Pajares & Graham's instrument measuring mathematical self-efficacy (1999), "How confident are you that you will get the correct answer?" a) before they had seen the problem, when all they knew was that they'd be working on a high school geometry problem, b) immediately after they finished reading the problem, and c) when they submitted an answer to the problem. (For that last measurement, I changed the tense of the question from future to past.) The participants rated their confidence on an eight-point Likert scale that ranged from "not confident at all" to "completely confident."

To determine how long it took each group to understand the goal of the problem, I recorded the time elapsed between a) the moment I first showed them the problem (by revealing a slide on a laptop) to b) the moment they recorded an indicator of their confidence before beginning work on the problem. During the interview, I asked my control participants to tell me at what point they understood the goal of the problem in order to account for the possibility that they had scanned immediately to the end of the problem, found the goal, and then doubled back to read the rest of the text.

I also asked the experimental group to describe how it felt not to have any information about the problem.

FINDINGS

1. Participants thought they were going to perform an exercise rather than solve a problem.

My participants expected a very different and much easier problem than the one I asked them to solve. They were all told in advance they'd be working on a high school geometry problem. When I asked them what kind of problem they were anticipating, they told me (variously):

- Brittany (a pseudonym): "I was expecting more of a 'Here's a shape, now do something with it.'"
- Ginny referred to the case study project for her student teaching program in which she shadowed a student to geometry class "a couple weeks ago and they were doing triangles and angles and stuff and like if you have a triangle and you know this angle and these two sides, how do you figure out the remaining angles and remaining sides or whatever. So that's what I was picturing as geometry and it was so easy when I was shadowing her."
- Maria and Mark were both participants in the experimental group and both reported anticipating something similar to what they'd find on "a standardized test" (Mark's words) or "the GRE" (Maria's).
- Mindy said she had seen some students "doing their geometry homework and they've been looking at parallel lines with the [transversal] and looking at the angles based on that. So alternate interior and things like that. So I was thinking more angles, more basic shapes, more 'If you have this angle, figure out these other angles.'"
- Brittany said she expected a problem with fewer steps. "Maybe they'd just stop after the first step," she said, "where it was just 'find the volume of this' or 'find the area of this' versus 'find the area and then you can do something with it.'"
- On her first look at the experimental treatment of the problem, Sunny whispered to her partner, "Since it was geometry, I was expecting shapes."

My interviews revealed that each participant had expected an exercise rather than a problem. Many had anticipated the application of a single skill in pursuit of a single numerical answer. The fact that the problem required the concerted application of several skills in an applied context confounded the expectations of every single participant.

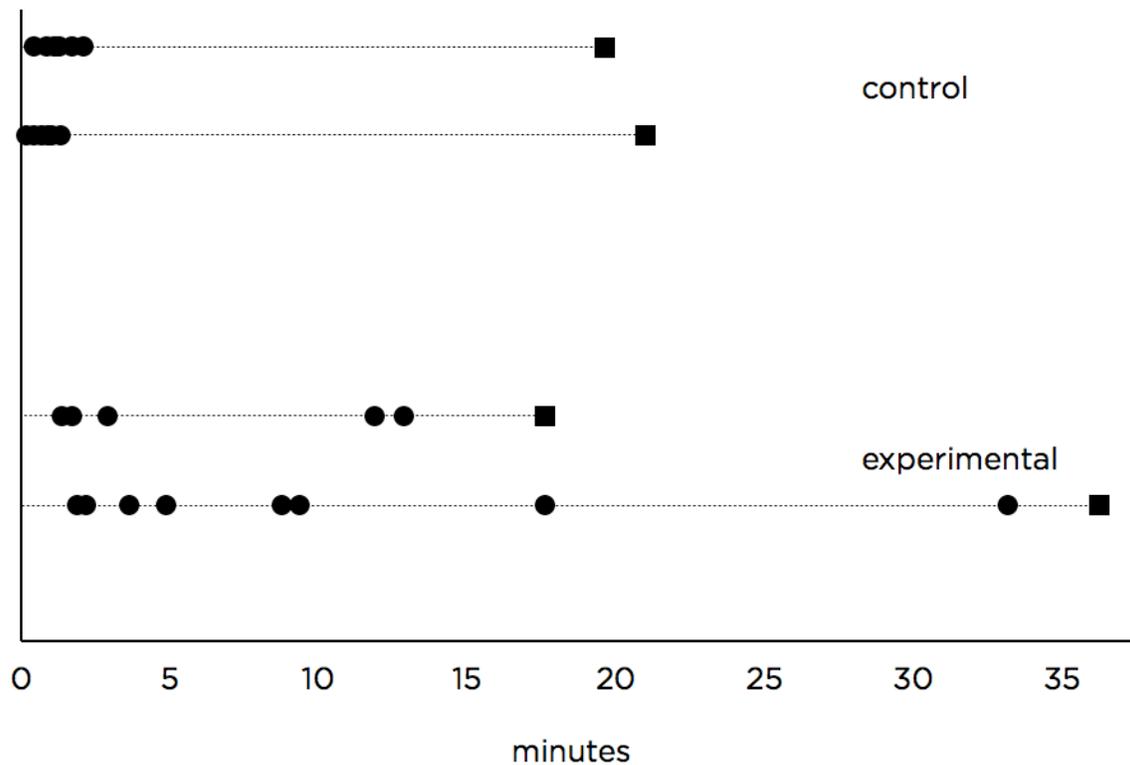
2. Participants in the experimental group chose to ask for information only when it served a strategy, not before. Participants in the control group didn't have that choice.

On average, the control pairs finished reading the released information 1:36 of the way through the duration of their problem solving sessions. The control pairs didn't ask for any more information, though it was explicit in the text of the problem they were allowed to.

I wasn't sure how the experimental group would request information, given the choice. If the control treatment was, in fact, preferable for them, we would expect similar statistics for the experimental group. But the experimental group stopped asking for more information an average of 21:57 of the way through the duration of their problem solving session. Given the choice, the experimental group waited an average of 4:48 from receiving one fact to requesting the next. It's clear that they were interrogating the problem space *continuously*, not just in one discrete moment at the start of the problem.

The precise release of information to the control and experimental groups is illustrated in Figure 2 below. The circles indicate released facts. The squares indicate the end of the problem solving session.

Figure 2: The timing of information release in both treatments.



3. *Participants in the experimental group asked for information until they had enough to implement a procedure. The control group had much less control over the release of information but developed its own protocols for managing it.*

The experimental pairs were eager, first, to find an overarching strategy for solving the problem. Mark, who had taken differential equations several years earlier, saw such a strategy quickly: 1:23 into the problem he asked for the volume of the tank and the flow rate of the hose.

It took Maria and Sunny, the other experimental pair, comparatively longer to compose a strategy. The absence of a strategy made Sunny uncomfortable. When Maria asked for the flow rate of the hose at 4:47 Sunny said, “Maybe we should first have a plan.” At 5:40, Sunny determined that plan: “Okay, speed of the water is 1.89

fluid ounces per second. If we knew the volume in fluid ounces, we would know the time. Does that make sense?” Up until the point they composed that strategy, Sunny and Maria interrogated the problem space for any information that seemed germane, even though they didn’t know how it would be used. Once they composed that strategy they stopped asking for information and began to implement its sub-procedures (eg. finding the area of the octagon) and didn’t ask for information again until after they completed each one.

Both experimental pairs seemed to have a tacit protocol for managing information. The highest-order goal for both pairs was finding a solution strategy for the entire problem, whether or not it could be implemented yet. The second-order goal was to gather sufficient information to implement a sub-procedure of their overall strategy. To confirm this protocol, I asked a pair why they didn’t just ask at once for all the information they’d need. Ben said, “We’d be overwhelming ourselves with information, all these different things. Like okay I was already asking about the height but we hadn’t even figured out [the area of the base] so I think we would definitely be getting ahead of ourselves.” At one point in the session itself, Mark illustrated this protocol by verbally restrained himself from asking for more information. At 12:58, he and Ben had enough information to find the area of a trapezoid. “And then now we just have to find the area of one of the trapezoids,” Mark said, “and then times that by two and add it together. But then we need the height of the container — whatever.” He cut himself off. “Let’s do one step at a time.”

The control pairs, meanwhile, had much less choice in the same matter of managing information. I released enough information to solve the problem to those pairs immediately, at the start of the problem. It was interesting, then, to watch those pairs try to re-assert some control over the release of information. Only one of the four control participants read the problem straight through, line-by-line. One of the others would write each new fact down as she read it. The other two integrated each new fact into a visual model of the problem, an ongoing sketch. I asked Mindy what purpose the sketch served. She said, “I think to keep track of different things so I’m not like overwhelmed by the amount of numbers or confusing units — things like that. If I can just write out, ‘Okay, here are the blocks I have.’”

4. *The text of the control problem functioned as an auxiliary problem solver, a silent partner who either helped or hindered the control participants.*

The experimental group was given nothing beyond the image of the problem and its goal statement. As we've seen, they had to come up with an overall strategy, then acquire information and procedures to service that strategy, and then implement those procedures. The same was true of the control group but it became clear that the extra text provided in their treatment was more than just a conduit for information about the problem. The extra text affected the strategy itself, almost as if that text were a third member of the control pairs.

This was most evident when the participants had to convert the result of their volume calculation from cubic centimeters to ounces. Both of the control groups successfully made that conversion. But the presence of the unit conversion information in the text of the control problem seemed to signal to the control participants that "you need to *do* something with this fact."

Ginny said, "The data that you included was sort of a clue. Like, 'Oh you're going to want to use all these numbers.'" Another control participant, Brittany, asked her partner if she "wanted to go back and make sure we got everything," before scanning back through the text to note that every fact had been used somewhere in the solution.

Boaler describes this silent partner in *Experiencing School Mathematics* (1997, p. 46) as a composite of "cues," "codes," and "implicit norms," all of which are external to a student's understanding of mathematics. She describes students who selected problem-solving strategies that emulated the last strategy the teacher modeled at the chalkboard. The students expected later problems in a set would require a more complicated application of earlier strategies. The students expected to use every number given in the text of a problem in their solutions.

This silent partner could also be a hindrance. Ginny initially mistook the unit conversion as the flow rate of the hose. Then she re-read those lines and eventually implemented the conversion correctly. The given information helped and hindered the control participants in ways that were difficult to predict.

I realized that releasing facts to the control participants in the text of a problem imposed a question on them that further distorted their problem solving: "What is the person who wrote this problem telling me by including these particular facts?" Whether or not the presence of those facts helped or hindered my control participants, it confounded my ability to assess their problem solving accurately.

I saw no such trouble in the experimental group. Maria and Sunny decided they needed to convert from one measure of volume to the other after noticing the different units on their volume measurement and the flow rate of the hose. They made this observation on their own, without the help of this silent partner in the text of the problem, which made it easier to attribute that step to their effective implementation of a strategy.

5. Both control pairs solved the problem correctly. Only one of the two experimental pairs solved the problem correctly.

Ben and Mark, an experimental pair, neglected to convert cubic centimeters to ounces and solved the problem incorrectly.

6. Participants in the experimental group understood the goal of the problem 1:07 minutes sooner than the participants in the control group.

This finding almost goes without saying. The goal of the problem is stated explicitly in the first sentence of the experimental treatment while five sentences and six facts precede the goal in the control treatment. It took the control pairs an average of 1:36 to encounter the goal of the problem while the experimental group understood the goal of the problem after an average of 29 seconds.

I wondered, though, if the control pairs had scanned quickly to the end of the problem, found the goal, and then doubled back to read the information. I asked after this possibility in my interview with the control group and they reported that they read each sentence in order.

7. On average, the experimental pairs took 6:54 minutes longer than the control group to finish the problem.

This makes the same kind of intuitive sense as the finding that it took the control group longer to *start* the problem. The control group had more text to read at the start of the problem. The experimental pairs had to determine they needed that information over the course of the problem and then ask for it. It's impossible to attribute the entire difference in duration to treatment effects, but it's worth noting that the experimental group took on average 26:55 minutes to finish the problem while the control group took 20:01 minutes.

8. The experimental and control groups were equally confident they'd be able to find the correct answer before they had seen the problem. After they saw the problem and at the end of the problem, the experimental group was less confident than the control group.

All the participants reported higher confidence as they completed sub-procedures and neared a solution. Both the experimental and the control group reported increases in confidence as (eg.) they found the height of a trapezoid or they calculated the area of the base or they found the volume of a prism. Both groups also started with similar median self-ratings. (See Table 1 and Figure 3.) Once they saw the problem, though, the control group's median self-rating increased slightly while the experimental group's declined by almost the same amount. As mentioned, the confidence of both groups increased thereafter but the control group's rate of change was greater than the experimental group's.

It is difficult to draw causal or even correlational claims from such a small sample but it's worth referring again to the text's service as an auxiliary problem solving partner. Ginny, a control group participant, noted that the information given was a clue. Brittany, also in the control group, re-read the text of the problem at the end of her solution to "make sure we got everything." Both of those remarks, again, are indicated of Boaler's

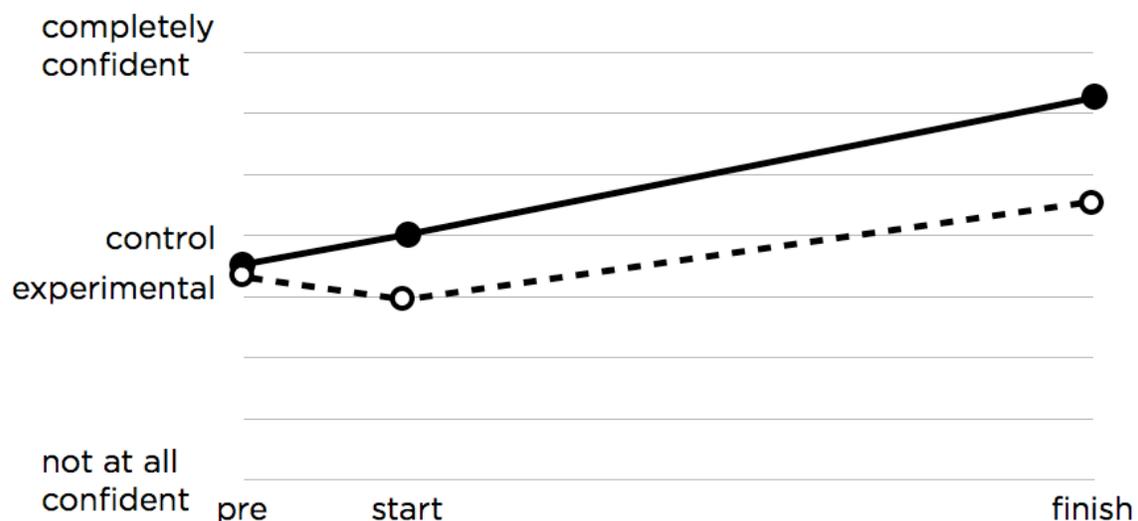
cues, codes, and implicit norms (1997) the presence of which might have increased the control group's final confidence rating.

Mark, a participant in the control group, attributed his lack of confidence at the end of the problem to his uncertainty about the units of the problem. "Yeah if I didn't start psyching myself on that I probably maybe would have jumped a little higher."

Table 1: Median confidence ratings on an eight-point Likert scale before, at the start of, and at the end of the problem solving session.

group	pre	start	post
control	4.5	5	7.25
experimental	4.25	4	5.5

Figure 3: A graph of both groups' median confidence ratings.



10. Higher-achieving math students didn't like the experimental treatment. Lower-achieving math students did.

Mark had taken differential equations three years earlier while Ben, his partner, had only taken statistics. Sunny was similar to Ben, having taken statistics to satisfy an undergraduate major requirement. But relative to her partner, who hadn't taken any math courses since her sophomore year of high school sixteen years earlier, Sunny was more like Mark. Both Sunny and Mark were markedly higher achieving in mathematics than their partners. Both Sunny and Mark liked the experimental treatment less than their partners. Both Sunny and Mark expressed *comfort and security* in seeing information in the text of a problem. Mark said, "In high school what helped me breeze through math was I was very good at knowing, when they were giving me certain numbers, I know what to do with them because I could memorize the equations."

Sunny also positioned the experimental treatment against her geometry experience in high school. "I guess I'm used to having a textbook with shapes and even if it was a picture that simulated real life there would be labels of numbers and so I would have all the information I knew I would need to solve a problem in front of me." She also said she liked having "something to look back to for a formula."

Meanwhile, Ben said that just seeing the picture was "intriguing" to him. "I want to figure out how to answer this," he said. "It makes me interested moreso [than if the information were already released] to get into it."

Maria remarked that seeing the water tank absent information or facts made it easier to relate to her own life. "I saw the filling something up with water and that's something I do everyday — when I'm filling up my Brita with water," she said. "I knew we would need [the information] I guess eventually. I just liked the idea of like even for myself saying 'how long do I think it would take to fill that up?'"

Mark and Sunny's preference for the control treatment calls into a question our definition of mathematics and the outcomes we prioritize for mathematics students. Both participants recall Boaler's cues, codes, and implicit norms. Sunny attributes her success in math to her ability to decipher the codes inherent to the tasks she was given. Mark explicitly cites *memorization* as a skill that was essential his success in mathematics as a student.

These participants' definitions of the successful study of mathematics compare unfavorably to the definitions posed by the reform documents cited earlier in this article. Ball (2003) describes the five strands of mathematical proficiency. None of those strands include "memorization." Instead, Ball and her panel ask proficient students for reasoning that is "adaptive." They ask for procedural fluency but clarify that it should be "flexible" fluency, the kind precluded by the memorization of a system of cues, codes, and implicit norms.

DISCUSSION

We have observed a particular distortion in how publishers treat applied mathematics in their textbooks. While applied mathematicians begin their problems with a goal absent the information essential to completing that goal, applied mathematics problems in textbooks present both at the same time. This study has tried to determine the cognitive and affective difference between contrasting treatments of applied mathematics problems.

When Thevenot, et al., (2007) moved the position of a word problem's question from the end of a problem to its beginning, they saw unequivocally positive results for both low- and high-achieving students. The results of this study, by contrast, varied with the participants' prior experience with mathematics and require a closer analysis of the treatment's costs and benefits.

On the one hand, the distortion seems to make students more confident in their ability to get a correct answer. We've seen that the presence of information in the text of the problem functions for students as a checklist. At the end of the problem, if they haven't used all the information given, they realize they may have made a mistake and look back at their work. The information also serves as an extra problem-solving partner. The presence of a unit conversion from cubic centimeters to ounces alerts students that, at some point, they should convert cubic centimeters to ounces.

While confidence is desirable, it's far from certain that this kind of confidence serves the best interests of our students. Students would be even more confident in their ability to get a correct answer, after all, if publishers wrote lots of hints in the

margin or simply gave students the correct answer. It's possible the control group's confidence rating reflects the fact that their work was simpler and less ambiguous than the work of the experimental group.

On the same hand, speaking in favor of the distortion, the evidence suggests that students may get the correct answer more often and take less time to get it than if the distortion were corrected and students had to determine the information essential to the problem on their own. These are, again, qualified recommendations in favor of the distortion. We have seen evidence that the released information functions as a silent problem solving partner, expending effort on behalf of our students, making it easier and faster for them to find an answer correctly. If the control participants could achieve the same gains in accuracy and speed while expending the same effort as the experimental participants, we would have grounds for preserving the distortion. As it is, the control participants did less work.

Arguments in favor of correcting the distortion are more convincing, though not without qualification themselves. We found that problem solvers, when given the choice, select a protocol for managing information that mimics the protocols of applied mathematicians. The experimental participants asked for information when they needed it, not before. By releasing all the information at the same time as the goal, we force students to implement artificial protocols they'll use less often when they apply mathematics to their own lives, like the visual sketch Mindy created to manage the release of information she didn't yet know she needed.

Moreover, we know it's important that students know the purpose of their work. Every second that elapses before students understand the point of a problem is costly. Control group participants had more to read and longer to wait before they knew the point of their work than the experimental participants who understood the point of the problem almost immediately. That difference may be insignificant to graduate students, but the difference may become more pronounced and its effects more debilitating for English Language Learners who will take even longer to parse text for meaning. We have a case here for correcting the distortion on the grounds of equitable access to challenging mathematics for all students.

It's true also that the experimental participants were less confident than the control participants throughout their problem solving session, but it's unclear what would happen over time if we corrected this distortion of applied mathematical problem solving. The experimental treatment was foreign to my participants, all of whom had grown accustomed to the distortion of applied mathematics over decades of primary, secondary, and undergraduate mathematics, all of whom had expectations that were stymied. It's an open question whether or not they'd grow progressively more confident the more often they had to ask themselves, "What information do I need in order to answer this question?"

If we're serious about implementing applied mathematical problem solving in the United States, if this isn't another fad or another period on the pendulum of education reform, we must consider the long-term effect of curricula, not just the short-term effect indicated by studies like this one. This study has attempted to define a distortion of applied mathematical problem solving and determine its affective and cognitive consequences for mathematical problem solvers, but it's been less assertive about a solution.

Solving the distortion requires an understanding of its source and the stakeholders interested in preserving it. The distortion exists, in one sense, because it makes students more confident and it increases the probability they'll successfully complete applied problems. Thus, teachers are sympathetic to the distortion because teachers will suffer the immediate consequences of students who are unconfident or disruptive. It's in their interest to preserve a system that keeps students confident and productive, whether or not that confidence is misplaced and whether or not the students could be even more productive.

Publishers are sympathetic to the distortion also, in large part because the experimental treatment of this study would be very difficult to implement in print, which is the predominant delivery medium of math curriculum. What would the experimental treatment look like on paper? Would one page ask students to consider the information they need to solve a problem, information that the publisher would reveal on some subsequent page? Math textbooks are already notoriously lengthy, heavy, and expensive. Correcting the distortion in print could prove costly in every way. But the

delivery of math curriculum is shifting slowly, along with the rest of the publishing industry, towards digital platforms, where an extra page or frame costs nothing, and the marginal weight of another bit is zero. It's in this medium that we may find the opportunity to correct a longstanding distortion and allow students to experience applied mathematics as it's practiced by applied mathematicians.

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APPENDICES

Appendix A — Information Cards

1. The water tank is in the shape of a prism.
2. The base of the prism is a regular octagon. [diagram]
3. A regular polygon has congruent sides and angles.
4. A prism has two identical bases and a height between them.
5. The height of the tank is 36.5 centimeters.
6. The side length of the regular octagon is 12.4 centimeters. [diagram]
7. The flow rate of the hose is 1.89 fluid ounces per second.
8. The distance from one side of the octagon to the other is 29.9 centimeters.
[diagram]
9. There are 29.57 cubic centimeters in 1 fluid ounce.
10. The hypotenuse of a right triangle is the side opposite the right angle. [diagram]
11. The interior angle of a regular octagon measures 135° . [diagram]

Appendix B — Procedure Cards

1. To find the volume of a prism, multiply the area of its base by its height.
2. To find the area of an octagon, divide it into smaller shapes or build a square around it.
3. To find the area of a trapezoid, find the average of its bases and multiply by the height. [diagram]
4. To find the area of a triangle, multiply its base by its height and divide by two.
[diagram]
5. To find the area of a rectangle, multiply its base by its height. [diagram]
6. To find the area of a square, multiply one side by itself. [diagram]
7. To find the time it takes to fill up a solid with a hose, divide the volume of the solid by the flow rate of the hose.

8. To find the leg of an isosceles right triangle from its hypotenuse, divide it by 1.4.
[diagram]
9. The height of an isosceles triangle, its base, and its base angle are related by this formula: $\tan(a) = 2h/b$. [diagram]

Appendix C — Content Logs